

Probing the geometric nature of particles mass in graphene systems

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Abstract. According to undulatory mechanics, the Compton periodicity, which is the intrinsic proper-time recurrence of a wave function, determines the mass of the corresponding elementary particles. This provides a geometric description of the rest mass which can be consistently applied to derive the effective mass spectrum and electronic properties of the elementary charge carriers in carbon nanotubes and other condensed matter systems. The Compton periodicity is determined by the boundary conditions associated to the curled-up dimension of carbon nanotubes or analogous constraints of the charge carrier wave function. This approach shows an interesting interplay between particle physics and relativistic space-time, as well as analogies with the Kaluza-Klein theory and Holography.

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Introduction

The origin of particles mass represents one of the most challenging research topics in both experimental and theoretical physics. It is for instance the main task of modern particle collider experiments such as LHC. On the theoretical side, the milestone of our understanding on the origin of particle's masses is the Higgs mechanism. Though the recent observation of a Higgs-like boson at the LHC, the problematics of the Higgs mechanism continue to motivate studies of various extensions of the standard model (extra-dimensions, supersymmetry, strong interacting models, etc), and even string theory (Regge trajectories, Veneziano amplitude, Anti de Sitter / Conformal Field Theory, also known as AdS/CFT correspondence, etc). All this represents huge efforts for the scientific community. Nevertheless, interesting aspects about the origin of particle masses, as well as other fundamental properties of elementary particles, can also be indirectly tested in a very accessible way through graphene physics, as also mentioned in recent papers [1,2,3,4]. Graphene allows us to simulate and manipulate (low dimensional) space-time geometries, and therefore to reproduce in test-tubes some of the fundamental space-time structures of high energy physics theories. A typical example is fact that the Dirac equation and the pseudo-spin emerges geometrically from the graphene sub-lattice structure [5,6].

The aim of this paper is a description of effective particles mass and energy bands in terms of simple geometric considerations. The low energy excitations of Elementary Charge Carriers (ECCs) in a graphene layer behave as massless particles. Nevertheless, in non trivial graphene structures such as Carbon Nanotubes (CNs) and bilayer graphene (BL), particles with a finite effective mass are generated. The origin of this effective mass can be easily interpreted in terms of the proper-time quantum recurrence prescribed by the wave-particle dualism for massive particles. As discovered by de Broglie [7], elementary particles are “periodic phenomena” whose recurrences determine their kinematics through the Planck constant. From this follows that the mass is related to a rest recurrence, namely the Compton periodicity, also known as the de Broglie internal clock (this corresponds to the *zitterbewegung*, i.e. “trembling motion”, for Dirac particles [8]). Indeed, as also pointed out by Penrose [9], this rest recurrence of massive elementary particles suggests that *every isolated particle is a relativistic reference clock* [1], that is “a clock directly linking time to a particle's mass” [10]. In graphene physics the quantum recurrences are directly determined by the geometry of the effective space-time associated to the lattice configurations. The diameter of the curled up dimension of CNs corresponds to the Compton length of the system. This fixes the rest recurrence and thus the ECCs mass scale. In [4] we have used similar arguments to derive the fundamental phenomenology of superconductivity (and a semiclassical description of the related electromagnetic symmetry breaking) and gap opening with particular reference to CNs.

A novel semiclassical quantum formalism encoding the quantum recurrence of elementary particles directly into space-time geometrodynamics has been recently proposed in [1,2,3,11]. For its geometric description of quantum dynamics, this formalism has particularly powerful applications in graphene systems. The main results of this paper is an intuitive derivations of electronic properties of ECCs in CNs and related condensed matter systems in terms of geometric space-time considerations. Our approach reveals fundamental correspondences between condensed matter and particle physics, summarized in Tab.(1), with possible insights on the fundamental nature of relativistic space-time, see [1]. We will also point out that the quantum behaviors of ECCs in CNs can be regarded as the Holographic dual of the Kaluza-Klein modes [12]. This correspondence has indeed an explicit interpretation in terms of the Compton periodicity associated to the curled up dimension of CNs [1,3,11,13].

1 Effective mass and Compton periodicity

The quantum recurrence of the ECCs wave function in a graphene layer is determined by the periodicities of the honeycomb lattice. By assuming the base vectors $\mathbf{a}_{1,2} = (\sqrt{3}, \pm 1)a/2$, where $a \simeq 2.46 \text{ \AA}$ is the lattice spacing, the periodicity of the 2D effective graphene space-time is $\mathbf{r} \equiv \mathbf{r} + \mathbf{a}_1 n + \mathbf{a}_2 m$, with $n, m \in \mathbb{N}$. Thus the ECCs wave function fulfills the following Periodic Boundary Conditions (PBCs):

$$\psi(\mathbf{r}) \equiv \psi(\mathbf{r} + \mathbf{a}_1 n + \mathbf{a}_2 m). \quad (1)$$

According to Bloch's theorem, these PBCs determine the behaviors of the ECCs in the graphene layer.

In undulatory mechanics, indeed, the energy $\bar{E}(\bar{\mathbf{p}})$ and momentum $\bar{\mathbf{p}}$ of a particle is determined by the time and spatial recurrences through the de Broglie - Planck - Einstein relations:

$$\bar{E}(\bar{\mathbf{p}}) = \frac{2\pi\hbar}{T(\bar{\mathbf{p}})}, \quad \bar{p}_i = \frac{2\pi\hbar}{\lambda_i}. \quad (2)$$

with $i = 1, 2$ in the case of the effective (2D+1) space-time defined on the graphene layer. In other words we can say that, according to the wave-particle duality, the energy-momentum and the space-time recurrences are two faces of the same coin.

As a consequence of the honeycomb lattice periodicity (1), the low energy propagation of the ECCs in a single layer graphene is characterized by the proportionality of the spatial and temporal recurrences of the wave function: $\lambda(\bar{\mathbf{p}}) = v_F T(\bar{\mathbf{p}})$. Indeed this is equivalent to say that the ECCs have the dispersion relation typical of a massless particles $\bar{E} = v_F |\bar{\mathbf{p}}|$, being the speed of light c replaced by the Fermi velocity v_F . In this paper, for the sake of simplicity, we will not consider the pseudo-spin degrees of freedom of the ECCs.

Graphene also suggests a geometrodynamical mechanism at the base of the generation of masses of elementary particles. The geometry of CNs is determined by a compactification vector \mathbf{C}_h , which describes the compactification of the curled up dimension in the effective graphene space-time. Its modulo C_h is the circumference of the CNs. The CN effective space-time resulting from this compactification is thus 1D. The curled-up dimension of CNs constrains the ECCs wave function to fulfill PBCs which in the coordinate system of the graphene lattice are

$$\phi(\mathbf{r}) = \phi(\mathbf{r} + \mathbf{C}_h). \quad (3)$$

In this section we neglect effects associated to the discrete lattice structure. This will be discussed in sec.(3).

In CNs, the compactified spatial dimension induces to the ECC an effective Compton periodicity with respect to the effective motion long the CN, i.e. a recurrence in the world-line parameter of CN effective 1D space-time: $\bar{\psi}(s) = \bar{\psi}(s + C_h)$. This can be seen by considering a ECC moving at velocity v_F along the CN curled up dimension in order to have a cyclic motion in this direction and to have zero velocity along the axial direction¹. This means that ECCs at rest in the CNs 1D effective space-time, namely $p_{\parallel} = 0$, have a residual intrinsic rest recurrence which thus corresponds to an intrinsic proper-time recurrence or equivalently to a world-line recurrence in the effective space-time, [1,2,3,11]. In terms of the effective proper-time coordinate $\tau = s/v_F$ of the CN 1D space-time, the PBCs (3) of the motion along the radial direction can be written as

$$\bar{\psi}(\tau) = \bar{\psi}(\tau + T_C)$$

where $T_C = T(p_{\parallel} = 0) = \frac{C_h}{v_F}$ is the Compton time period, or equivalently as a world-line recurrence of period C_h which represents the Compton wavelength of the system. In turns, the mass scale of the ECCs is determined by this proper-time periodicity according to the Compton relation

$$\bar{m} = \frac{2\pi\hbar}{C_h v_F} = \frac{2\pi\hbar}{T_C}. \quad (4)$$

¹ As we will describe, as a consequence of the lattice structure, in some CN geometries is not possible to have a purely radial motion.

Indeed in undulatory mechanics the mass (rest energy) corresponds, through the Planck constant, to a rest periodicity of the wave function, i.e. to a proper-time recurrence or equivalently to a world-line recurrence. This also means that the ECCs in single layer graphene, being massless, have an infinite Compton periodicity (i.e. infinite CN compactification length). This describes the effective mass scale in CNs in agreement with experimental observations. In sec.(3) we will derive the exact mass spectrum in the specific CNs configurations by considering the corresponding lattice geometry.

The same arguments can also be extended to describe the generation of effective mass in other graphene systems. For instance, in graphene bi-layer the role of the rest recurrence is determined by the distance d of the two layers. Through electromagnetic interaction, the ECCs among the two layers results strongly hybridized. ECCs at rest in the planar direction have a correlation among the layers Σ (the perpendicular direction is denoted by x_\perp). In analogy with the rest periodicity in CNs, this fixes the boundary conditions for the wave function $\partial_\perp \psi(x_\perp)|_\Sigma = 0$ (i.e. a particle in a box or a vibrating string in which Dirichlet BCs are replaced by Neumann BCs). The Compton length, i.e. the analogous of the compactified dimension of the CN, is in this case given by the distance among the layers and the spatial dynamics of the ECCs is bi-dimensional [14,15]. This separation d is of the order of $\sim 10^{-10}\text{m}$. If compared with the Compton length of the electron $\sim 10^{-12}\text{m}$ we find that the effective mass of the ECCs in graphene bilayer is $\bar{m} \sim 10^{-2} \times m_e$ as confirmed by observations². More complex graphene systems such as multiwalled CNs are described by the combination of these two fundamental cases so that, for example, the mass spectrum is characterized by two fundamental mass scales.

	Graphene	Nanotubes	Relativistic space-time
Dimensions	2D+1	1D+1	3D+1
Speed of Light	v_F	v_F	c
Periodicity	$\psi(\mathbf{r}) \equiv \psi(\mathbf{r} + \mathbf{a}_1 n + \mathbf{a}_2 m)$ (+ $\partial_\perp \psi(x_\perp) _\Sigma = 0$ for BL)	$\psi(\mathbf{r}) = e^{i2\pi\alpha} \psi(\mathbf{r} + \mathbf{C}_h)$ or $\phi(x_\mu^*) = e^{i2\pi\alpha} \phi(x_\mu^* + v_F T_\mu^*)$	$\phi(x^\mu) = e^{i2\pi\alpha} \phi(x^\mu + c T^\mu)$
Compton periodicity	$T_C = \infty$ ($T_C = d/2\hat{c}$ for BL)	$T_C = \frac{C_h}{v_F}$	$T_C = \frac{\Lambda_C}{c}$
Mass Scale	$\bar{m} \sim 0$ ($\bar{m} \sim \frac{\hbar c}{d}$ for BL)	$\bar{m} = \frac{\hbar}{ \mathbf{C}_h v_F} = \frac{\hbar}{T_C v_F^2}$	$\bar{m} = \frac{\hbar}{\Lambda_C c} = \frac{\hbar}{T_C c^2}$
Dispersion relation	$\bar{E}^2(\bar{\mathbf{p}}_\parallel) \simeq \bar{p}_\parallel^2 v_F^2$	$\bar{E}^2(\bar{\mathbf{p}}) = \frac{(2\pi\hbar)^2}{T^2(\bar{\mathbf{p}}_\parallel)} \simeq \bar{m}^2 v_F^4 + \bar{p}_\parallel^2 v_F^2$	$\bar{E}^2(\bar{\mathbf{p}}) = \frac{(2\pi\hbar)^2}{T^2(\bar{\mathbf{p}})} = \bar{m}^2 c^4 + \bar{\mathbf{p}}^2 c^2$
Phase harmony	$\bar{\mathbf{p}} \cdot (\mathbf{a}_1 n + \mathbf{a}_2 m) = 2\pi\hbar$	$\bar{\mathbf{p}} \cdot \mathbf{C}_h = \bar{p}_{*\mu} T_\mu^* = 2\pi\hbar$	$\bar{\mathbf{p}}_\mu T^\mu \equiv 2\pi\hbar$
space-time recurrence	$0 \equiv \frac{1}{T^2(\bar{\mathbf{p}})} - \sum_i \frac{v_F^2}{\lambda_i^2(\bar{\mathbf{p}})}$	$\frac{1}{T_C^2} = \frac{1}{T_\mu^* T_{*\mu}} = \frac{1}{T^2(\bar{\mathbf{p}}_\parallel)} - \frac{v_F^2}{\lambda_\parallel^2(\bar{\mathbf{p}}_\parallel)}$	$\frac{1}{T_C^2} = \frac{1}{T^\mu T_\mu} = \frac{1}{T^2(\bar{\mathbf{p}})} - \sum_i \frac{c^2}{\lambda_i^2(\bar{\mathbf{p}})}$
Energy spectrum	Bloch theorem ...	Cont: $E_n(p_\parallel) = (n + \alpha) \sqrt{\bar{m}^2 v_F^4 + \bar{p}_\parallel^2 v_F^2}$ ZZ: $E_n^2(\bar{p}_\parallel) = \bar{m}^2 v_F^4 \frac{N^2}{3\pi^2} (1 + 2 \sin \frac{\pi n}{N})^2 - \frac{2}{3} \bar{p}_\parallel^2 v_F^2 \cos \frac{\pi n}{N}$	$E_n(\bar{\mathbf{p}}) = (n + \frac{1}{2}) \sqrt{\bar{m}^2 c^4 + \bar{\mathbf{p}}^2 c^2}$ (free particle)
Rest spectrum	none in the low energy approximation	Cont $m_n = (n + \alpha) \bar{m}$ ZZ: $m_n = \bar{m} \frac{N}{\pi} \sin \frac{\pi n}{N}$ AC: $m_n = \bar{m} \frac{N}{3\pi} (1 + 2 \cos \frac{\pi n}{N})$	$E_n(0) = (n + \frac{1}{2}) \bar{m} c^2$ (not observable for free particles)
Spectral Density		$\rho(E) = \mathcal{R} \sum_n \frac{ E_n(p_\parallel) }{\sqrt{E_n^2(p_\parallel) - \bar{m}_n^2 v_F^2}}$	$Im\Pi(p^2) = \sum_n \frac{F_n^2(p^2)}{p^2 - \bar{m}_n^2 c^2}$ strongly interacting (e.g. QCD)

Table 1. The table summarizes the analogies between the behavior of ECCs in the effective space-time of graphene systems and the behavior of elementary particles in relativistic space-time, emphasizing the rule in of the quantum recurrences in both cases. The graphene systems reported are single layer graphene, bilayer graphene (BL), and CNs in the continuum limit (Cont), in the Zigzag (ZZ) and in the Armchair (AC) configurations.

A similar geometric description of the mass generation can be generalized to elementary particles. To see this we consider that the speed of light c relates the mass to the rest energy according to $\bar{E}(0) = \bar{m}c^2$. On the other hand, in quantum mechanics, the Planck constant relates the energy $\bar{E}^2(\bar{\mathbf{p}})$ of a particle in a generic reference frame denoted by $\bar{\mathbf{p}}$ to the relativistic time recurrence $T(\bar{\mathbf{p}})$ in that reference frame according to (2). Indeed, by considering the de Broglie relation for a rest particle, the Planck constant associates the particle's mass \bar{m} to a proper-time periodicity

² For a more accurate evaluation it is necessary to consider the reduced speed of light \hat{c} in the motion among the two layers.

$T_C = T(0)$. Similarly to (4), the Compton periodicity for ordinary particles is

$$T_C = T(0) = \frac{A_s}{c} = \frac{2\pi\hbar c^2}{\bar{m}} = \frac{2\pi\hbar}{\bar{E}(0)}. \quad (5)$$

Equivalently we can say that, according to the wave-particle duality, every massive particle is characterized by recurrences along its world-line $s = c\tau$ whose period is the Compton wavelength of the particle $\lambda_s = T_C c$. A full description of this idea is given in [1, 2, 3, 11]. The heavier the mass, the faster the characteristic period of the internal clock. Even considering a light particles such as the electron, this Compton temporal period is extremely fast, about $T_C \sim 10^{-21}$ s. This is fast even if compared to the characteristic periodicity of the Cs133 clock, about 10^{-10} s or to the present resolution in time which is of the order of 10^{-17} s (close to the attoseconds scale). On the other hand, a massless particle such as the photon has infinite proper-time periodicity, i.e. infinite Compton length (we say that the internal clock of a massless particle is frozen). CNs have therefore the important property to effectively rescale the Compton periodicity of elementary particles to values directly observable experimentally. Since the Fermi velocity in graphene is about $v_F \sim 10^6$ m/s and the typical circumference of CNs is of the order of $C_h \sim 10^{-9}$ m, the rescaled Compton periodicity in CNs is $T_C \sim 10^{-15}$ s, which today is experimentally accessible [16]. Besides the experimental interest, this direct observation of the cyclic dynamics of elementary particles is relevant to understand foundational aspects of quantum mechanics such as the *zitterbewegung* [8], the emergence of the wave-particle duality from space-time geometrodynamics or related aspects [1, 2, 3, 11]. Finally, a good modelization of the ECCs cyclic dynamics is important to have an efficient control CNs as ultrafast electronic devices, which are likely to have relevant applications in next hardware generation [17], and thus to have an efficient manipulation of the related electronic information.

2 Dispersion relation and space-time recurrence

The dispersion relation of ECCs in CNs leads to interesting geometric considerations about quantum particles space-time dynamics. For the sake of simplicity, here we consider the continuum graphene lattice limit.

Direct observations show that in the continuum limit the typical dispersion relation of the ECCs in a CNs is [18]

$$\bar{E}^2(\bar{p}) \simeq \bar{m}^2 v_F^4 + \bar{p}_{\parallel}^2 v_F^2 + \mathcal{O}(\bar{p}^3). \quad (6)$$

This approximates the dispersion relation of relativistic massive particles, provided that the speed of light c is replaced by the Fermi velocity v_F . In the effective space-time of CNs, the energy-momentum is a two-vector $\bar{p}_{*\mu}$ which can be inferred from the rest mass through Lorentz transformations: $\bar{m} \rightarrow \bar{p}_{*\mu} = \{\bar{E}/v_F, -\bar{p}_{\parallel}\}$ where $\bar{E} = \gamma_* \bar{m} v_F^2$, $\bar{p}_{\parallel} = \gamma_* v_{\parallel} \bar{m}$, $\gamma_* = (1 - v_{\parallel}^2/v_F^2)^{-1/2}$ and v_{\parallel} is the velocity of the ECCs along the CNs axis.

The geometric interpretation of (6) in terms of space-time recurrences is the following. The rest state is characterized by a proper-time recurrence T_C , see (5). Since $\bar{p}_{\parallel} = 0$, the ECCs at rest have an infinite spatial recurrence in the axial direction of the CN: $\lambda_{\parallel} = \infty$. We want now consider ECCs moving from their rest frame, for instance as a consequence of a voltage gradient along the axial direction. As the ECCs start to move along the CN, the non vanishing \bar{p}_{\parallel} corresponds to a finite spatial recurrence along the axis, $\lambda_{\parallel}(p_{\parallel}) = 2\pi\hbar/\bar{p}_{\parallel}$, namely (2) with $i = \parallel$. On the other hand, in analogy with the relativistic Doppler effect, the temporal periodicity turns out to be modulated with respect to the rest periodicity according the energy of the motion: $T(\bar{p}_{\parallel}) = 2\pi\hbar/\bar{E}(\bar{p}_{\parallel})$. The dispersion relation (9) implies that these space-time periodicities (in the continuum limit) are related by the constraint $\frac{1}{T_C^2} = \frac{1}{T(\bar{p}_{\parallel})^2} - \frac{v_F^2}{\lambda_{\parallel}^2(\bar{p}_{\parallel})}$, as can be easily inferred multiplying by $(2\pi\hbar)^2$. This means that $T(\bar{p}_{\parallel})$ and λ_{\parallel} transform as the components of a contravariant space-like tangent two-vector $T_{*}^{\mu} = \{T(p_{\parallel})v_F, \lambda_{\parallel}\}$. That is, T^{μ} can be derived by Lorentz transforming the rest recurrence: $v_F T_C \rightarrow v_F \gamma_* T - \gamma_* v_{\parallel} \lambda_{\parallel}/v_F$. The energy-momentum can be directly derived from T_{*}^{μ} according to the de Broglie—Planck—Einstein relation. By using the Lorentz transformation of T_C described above, this relation between energy-momentum and space-time recurrence can be equivalently written as a so-called phase-harmony condition

$$\bar{m} T_C v_F^2 \equiv 2\pi\hbar \quad \Rightarrow \quad \bar{p}_{*\mu} T_{*}^{\mu} \equiv 2\pi\hbar. \quad (7)$$

We now generalize this description to relativistic space-time and elementary particles, [1, 2, 3, 11]. Lorentz transformations imply that the mass \bar{m} of an elementary particle, if observed in a non inertial reference frame, determine the kinematical state of the particle: $\bar{m} \rightarrow \bar{p}_{\mu} = \{\bar{E}/c, -\bar{\mathbf{p}}\}$, where $\bar{E} = \gamma \bar{m}$ and $\bar{\mathbf{p}} = \gamma \mathbf{v} \bar{m}$. On the other hand, as noted in [1, 2, 3, 11], the Lorentz projection of the rest recurrence (5) into a generic reference frame determines the ordinary temporal and spatial quantum recurrence³ $T^{\mu} = \{T_C, \boldsymbol{\lambda}\}$ according to $cT_C \rightarrow c\gamma T - \gamma \boldsymbol{\beta} \cdot \boldsymbol{\lambda}$. As prescribed by the wave-particle duality, the space-time recurrence $T^{\mu}(\bar{\mathbf{p}})$ fixes through the Planck constant the local four momentum

³ These recurrences can emerge from microscopic structure of space-time such as in scale relativity [19].

of the particle \bar{p}_μ in the corresponding reference frame, according to (2) with $i = 1, 2, 3$ denoting the three spatial dimensions of ordinary relativistic space-time. Similarly to the case of CNs, the energy-momentum and the space-time recurrence of elementary particles are related by the phase-harmony condition

$$\bar{m}T_C c^2 \equiv 2\pi\hbar \quad \Rightarrow \quad \bar{p}_\mu T^\mu \equiv 2\pi\hbar. \quad (8)$$

This is a scalar product (the phase of a field is invariant) so that T^μ transforms as a contravariant tangent space-like four-vector [20, 1, 2, 3, 11] (\bar{p}_μ is a covariant tangent four-vector). We find that T^μ satisfies the reciprocal of the relativistic dispersion relation of the four momentum: $\bar{m}^2 = \bar{p}_\mu \bar{p}^\mu \Leftrightarrow \frac{1}{T_C^2} = \frac{1}{T^\mu} \frac{1}{T_\mu}$ ⁴. Indeed, similarly to the relativistic Doppler effect, the global temporal periodicity of elementary particles as observed from different reference frames is modulated according to the relativistic dispersion relation

$$\bar{E}^2(\bar{\mathbf{p}}) = \frac{(2\pi\hbar)^2}{T^2(\bar{\mathbf{p}})} = \bar{m}^2 c^4 + \bar{\mathbf{p}}^2 c^2. \quad (9)$$

3 Energy bands and quantum energy spectrum

In this paragraph we will consider the effect of the finite graphene lattice. Besides the lattice periodicity (1), the exact energy band structure of the ECCs in CNs is determined by the PBCs associated to the curled up dimension (3). The lattice definition of the compactification vector is $\mathbf{C}_h = N_1 \mathbf{a}_1 + N_2 \mathbf{a}_2$, with fixed $N_1, N_2 \in \mathbb{N}$.

The hexagonal lattice of graphene implies that some CNs configurations (those with $N_1 - N_2 = 3N \pm 1$ with $N \in \mathbb{N}$) are symmetric under rotations of $\pm \frac{2}{3}\pi$. This is the case of semiconducting CNs. This symmetry implies that the PBCs (3) have a twist factor

$$\bar{\psi}(\mathbf{r}) = e^{i2\pi\alpha} \bar{\psi}(\mathbf{r} + \mathbf{C}_h). \quad (10)$$

Indeed, for semiconducting CNs we have $\alpha = \pm \frac{1}{3}$, whereas the case of $\alpha = 0$, i.e. (3), refers to metallic CNs.

The PBCs (10) imply a quantization of the ECCs in analogy with the “particles in a box” or a vibrating string (provided that the Dirichlet BCs at the walls are replaced by PBCs). The ECCs wave function turns out to be a superposition of all the possible harmonic modes satisfying (10).

In terms of the coordinate system of the graphene lattice the quantization condition for the ECC energy-momentum coming from (10) is therefore

$$e^{i2\pi\alpha} e^{-\frac{i}{\hbar} \mathbf{p}_n \cdot \mathbf{C}_h} = 1 \quad \Rightarrow \quad \mathbf{p}_n \cdot \mathbf{C}_h = 2\pi\hbar(n + \alpha). \quad (11)$$

Alternatively the ECC evolution can be described in terms of the CN effective space-time coordinates $x_*^\mu = \{t, x_\parallel\}$. In a generic reference frame, the space-time periodicity T_*^μ of the ECCs on the CNs turns out to define the following PBCs for the wave function

$$\bar{\psi}(\mathbf{r}) = e^{i2\pi\alpha} \bar{\psi}(\mathbf{r} + \mathbf{C}_\tau) \quad \Rightarrow \quad \bar{\psi}(x_*^\mu) = e^{-i2\pi\alpha} \bar{\psi}(x_*^\mu + v_F T_*^\mu). \quad (12)$$

ECCs at rest with respect to the axial direction has proper-time periodicity T_C . When imposed as a constraint, such a rest periodicity implies a quantization of the rest energy, i.e. of the mass spectrum. The wave function in the rest frame can be therefore written as superposition of mass eigenstates $\bar{\psi}(\tau_*) = \sum_n A_n \exp[-im_n v_F^2 \tau / \hbar]$ whose eigenvalues are determined by the rest PBCs

$$\bar{\psi}(\tau) = e^{i2\pi\alpha} \bar{\psi}(\tau + T_C) \quad \Rightarrow \quad m_n c^2 T_C = 2\pi\hbar(n + \alpha). \quad (13)$$

In the continuum lattice the resulting mass spectrum is harmonic with a shift associated to the twist factor [18, 4]

$$m_n \simeq (n + \alpha) \bar{m} = (n + \alpha) \frac{2\pi\hbar c^2}{T_C}. \quad (14)$$

Indeed the fundamental band of semiconducting CNs has mass $m^* = \frac{2\pi\hbar c^2}{3T_C}$ whereas is massless in the metallic case.

⁴ This description is consistent as long as we want to deal with free particles. For a consistent description of interaction local modulations of T^μ must be considered as described.

The effective mass spectrum follows by considering that the periodicity is on a lattice ⁵. For Zigzag ($N_1 = N$ and $N_2 = 0$) and Armchair ($N_1 = N_2 = N$) CNs we have [21] respectively

$$m_n^2 \simeq \frac{(2\pi\hbar)^2}{T_C^2 v_F^4} \frac{N^2}{\pi^2} \sin^2 \frac{\pi n}{N}, \quad (15)$$

$$m_n^2 \simeq \frac{(2\pi\hbar)^2}{T_C^2 v_F^4} \frac{N^2}{3\pi^2} \left(1 + 2 \cos \frac{\pi n}{N}\right)^2. \quad (16)$$

A similar mass spectrum can also be observed in superlattice of quantum stripes where the rest periodicity for the motion parallel to the strips is determined by the gap L between the stripes (notice however that the ECCs have a non-relativistic dispersion relation and implies that the mass spectrum goes like n^2 , that is $m_n \propto n^2/L^2$) [22].

Finally, by considering the relativistic like dispersion relation of CNs (i.e. the modulations of spatial and temporal periodicities associated to the motion along the axial direction) we find, in the continuum limit, the energy bands

$$E_n^2(p_{\parallel}) \simeq (n + \alpha)^2 \frac{(2\pi\hbar)^2}{T^2(\bar{\mathbf{p}}_{\parallel})} = (n + \alpha)^2 (\bar{m}^2 v_F^4 + \bar{p}_{\parallel}^2 v_F^2). \quad (17)$$

The lattice version of this equation gives the effective energy bands of CNs. For instance, in the Zigzag configuration, we have [21]

$$E_n^2(\bar{p}_{\parallel}) \simeq \bar{m}^2 v_F^4 \frac{N^2}{3\pi^2} \left(1 + 2 \cos \frac{\pi n}{N}\right)^2 - \frac{2}{3} \bar{p}_{\parallel}^2 v_F^2 \cos \frac{\pi n}{N}. \quad (18)$$

As proposed in recent papers [1,2,3,11], the intrinsic space-time recurrence of elementary particles T^μ can be used as a constraint quantizing semiclassically particles. A particle constrained to have periodicity exhibits in fact a quantized spectrum in analogy with a “particle in a box”. That is, depending on the reference frame of a free particle, we can semiclassically quantize the wave function of elementary particles by imposing

$$\phi(x^\mu) = e^{-i2\pi\alpha} \phi(x^\mu + cT^\mu) \quad \Rightarrow \quad p_{n\mu} T^\mu = 2\pi\hbar(n + \alpha). \quad (19)$$

This is the relativistic analogous for an elementary free particle of the quantization condition (10), where we have introduced a twist factor for completeness⁶. It can be regarded as the generalization to free relativistic particles of the Bohr-Sommerfeld quantization prescription of close orbits.

The resulting semiclassical cyclic dynamics associated to T^μ are formally equivalent to the ordinary relativistic quantum mechanics of the particles. The correspondence has been proven for both the canonical and the Feynman formulations of quantum mechanics, [1,2,3,11]. The harmonics associated to the intrinsic recurrence corresponds to the quantum excitations of the elementary particles.

Notice that, by following the same line used to derive the energy bands in CNs, and considering only the temporal component of the space-time periodicity, the quantization of the energy spectrum associated to the time periodicity $T(\bar{\mathbf{p}})$ is the harmonic (quantized) spectrum. In the case $\alpha = 1/2$ it coincides with the quantized energy spectrum of an harmonic oscillator of period $T(\bar{\mathbf{p}}) = 2\pi/\bar{\omega}(\bar{\mathbf{p}})$: that is, $E_n(\bar{\mathbf{p}}) = (n + \frac{1}{2}) \frac{2\pi\hbar}{T(\bar{\mathbf{p}})} = (n + \frac{1}{2}) \hbar \bar{\omega}(\bar{\mathbf{p}})$. By considering the relativistic modulations (relativistic Doppler effect) of temporal periodicity $T(\bar{\mathbf{p}})$ associated to a generic reference frame denoted by $\bar{\mathbf{p}}$, see (9), the resulting energy spectrum is

$$E_n(\bar{\mathbf{p}}) = \left(n + \frac{1}{2}\right) \frac{2\pi\hbar}{T(\bar{\mathbf{p}})} = \left(n + \frac{1}{2}\right) \hbar \bar{\omega}(\bar{\mathbf{p}}) = \left(n + \frac{1}{2}\right) \sqrt{\bar{\mathbf{p}}^2 c^2 + \bar{m}^2 c^4}. \quad (20)$$

In other words we have obtained semiclassically the energy spectrum prescribed by ordinary second quantization for bosonic particles⁷. The normal ordering correspond to put $\alpha = 0$. This suggest that the vacuum energy can be interpreted semiclassically in terms of space-time geometrodynamics as a twist factor in the space-time periodicity.

By evaluating the energy spectrum (20) in the rest frame we obtain a quantized rest energy spectrum, i.e. a quantized mass spectrum $m_n := E_n(0)/c^2 = n\bar{m}$ (after normal ordering). At a first sight this result may appear unusual for free particles. Nevertheless it must be noticed that a free particle at rest (non relativistic limit) is a

⁵ When we pass from a continuum vibrating string of fundamental frequency $1/T$ to a chain of N masses and springs (string on a lattice) the harmonics spectrum modifies as $\omega_n = \frac{n}{T} \rightarrow \frac{N}{T\pi} \sin \frac{\pi n}{N}$.

⁶ For the wave function only the modulo has a physical meaning, so that the recurrence of a free elementary particle can be defined modulo a global twist factor. The case of interacting particles is given by local modulations of the space-time periodicity, [1,2,3,11].

⁷ Indeed, second quantization prescribes that every mode with angular frequency $\bar{\omega}(\bar{\mathbf{p}}) = \sqrt{\bar{\mathbf{p}}^2 c^2 + \bar{M}^2 c^4}/\hbar$ of a particle has a quantized energy spectrum $E_n(\bar{\mathbf{p}}) = (n + 1/2)\hbar\bar{\omega}(\bar{\mathbf{p}})$.

classical free particle (it has not quantum corrections). This means that in this classical limit the higher energy levels cannot be excited and thus the mass spectrum cannot be observed (in the quantum-relativistic limit the energy levels can be thought of as forming 'multiparticle states' or 'virtual particles', whereas in the non-relativistic limit the rest energy term, and thus the Compton periodicity, can be neglected). In CNs and similar condensed matter systems the ECCs are in a thermal bath and this implies that the higher mass eigenstates can be excited and observed. As we will discuss below, another example is given by strongly interacting systems such as QCD where actually a similar (deformed) mass eigenvalues can be directly observed and identified, for instance, with the hadrons (e.g. in the Regge theory $m_n \sim \sqrt{n\bar{m}}$).

4 Density of states and quantum correlator with comments about the Kaluza-Klein theory and Holography

Graphene physics can be used to test advanced theories of particles physics, such as Kaluza-Klein theory, Holography and dimensional compactifications.

Interesting analogies between in extra-dimensional theories and CNs have been pointed out in [21]. We have already discussed the mass spectrum of CNs, including its continuum lattice limit, and the corresponding energy bands (17, 18). From these follow the density of states, which can be in general expressed in terms of the mass eigenstates and (neglecting the pseudo-spin) written as, [18],

$$\rho(E) = \mathcal{R} \sum_n \int dk_{\parallel} d(k_{\parallel} - k_{\parallel n}) \frac{|E(\hbar k_{\parallel})|}{\sqrt{E^2(\hbar k_{\parallel}) - m^2 v_F^2}} = \mathcal{R} \sum_n \frac{|E_n(p_{\parallel})|}{\sqrt{E_n^2(p_{\parallel}) - m_n^2 v_F^4}}, \quad (21)$$

where $\mathcal{R} = \frac{\sqrt{3}a^2}{2\pi\hbar T_C v_F}$ and $k_{\parallel n}$ are roots of the equation $E - E_n(\hbar k_{\parallel}) = 0$. The mass eigenmodes have collective behaviors. The energy bands $E_n(\bar{p}_{\parallel})$ are in fact expressed as functions of the single parameter, namely the fundamental momentum \bar{p}_{\parallel} . In other words they form the quantum excitations of a single wave-packet.

As correctly pointed out by the authors of [21], the mass spectrum of CNs (14) in the continuum limit is formally equivalent to the mass spectrum of a Kaluza-Klein theory whose compactified length is $\Lambda_{KK} \equiv C_h$, plus a possible twist factor in the PBCs, see Hosotani or Scherk-Schwartz mechanism [23,24]. For instance, the lattice version of the Kaluza-Klein mass spectrum, see "dimensional (de)construction" [25,26], corresponds to the metallic CNs spectrum (15).

A fundamental difference between CNs and Kaluza-Klein theory (in 1D plus one extra-dimension) has to be however noticed. In the Kaluza-Klein theory the different mass eigenstates are independent classical particles of masses m_n , i.e. with unrelated momenta \mathbf{p}_n . Nevertheless, a collective description of the Kaluza-Klein modes is recovered in the holographic description of extra-dimensional dynamics. Indeed, according to the AdS/CFT correspondence, these individual Kaluza-Klein modes are dual to quantum excitations [27,12]. In other words, the energy bands of CNs are the holographic duals of the Kaluza-Klein modes and the curled up dimension of CNs corresponds to the holographic dimension of AdS/CFT. Without entering into the details of Holography and AdS/CFT, we may simply consider the example of AdS/QCD in which the holographic description of the Kaluza-Klein modes in a warped extra-dimension can be approximatively identified with the hadrons [26,28,29]. Indeed the hadrons can be thought of as quantum excitations of the same fundamental system (e.g. vibrating warped string) [30]. For instance, in the holographic formulation, the correlation function associated to the extra-dimensional theory, i.e. the Fourier transform of the spectral of Kaluza-Klein modes density [31,32], turns out to be

$$Im[\Pi(p^2)] = \sum_n \int dq^2 d(q^2 - m_n^2 c^2) \frac{F_n^2(p^2)}{p^2 - q^2} = \sum_n \frac{F_n^2(p^2)}{p^2 - m_n^2 c^2}, \quad (22)$$

where $F_n(p^2)$ are form factors. This can be compared with the density of states in CNs (provided that the parametrization is given in terms of E rather than p^2). In the case of warped extra-dimension such a spectral density qualitatively describe the two point function of hadrons in QCD.

The origin of these analogies between the particles dynamics in CNs and extra-dimensional theories must be found in the recurrence of world-line parameter prescribed to massive particles by undulatory mechanics. The details of this description have been discussed in [1,3,11,13]. Here it is sufficient to give simple considerations about the metrics. The metric of the Kaluza-Klein theory is $dS^2 = dx^2 - dy^2$. The original Kaluza-Klein theory involves extra-dimensional massless fields, so that their dynamics are in the extra-dimensional light-cone, namely $dS^2 \equiv 0$. Moreover the extra-dimension y is supposed to be cyclic with recurrence Λ_{KK} . Thus, through the equations of motion, the metric can be formally written as $dy^2 = dx^2$. In this way we see that the extra-dimension plays the role analogous to that of a cyclic world-line parameter s in the lower dimensional space-time theory, $ds^2 = dx^2$. However, by assuming explicitly a cyclic

world-line as in CNs, we find that the eigenstates have a collective behavior (similarly to the wave-packet associated to a particle constrained to have intrinsic periodicity or to be “in a box”) rather than the behavior of individual particles as in the Kaluza-Klein theory, and C_h fixes the mass scale through the Compton relation, in agreement to Holography and AdS/CFT where a compact holographic dimension determines an effective mass scale and thus the breaking of the conformal invariance [26,28]. To emphasize the analogies with the dynamics associated to a cyclic extra-dimension, we say that the world-line in CNs acts as a “virtual extra-dimension”. The concept of “virtual extra-dimension”, its implications and the related interpretation of Holography are given in [1,3,11,13].

5 Conclusions and remarks

In recent literature there is a growing interests in studies linking the effective graphene space-time to fundamental aspects of particle physics. One of the most studied aspects is for instance the generation of the (pseudo) spin degrees of freedom from the triangular sub-lattice structure of the graphene honeycomb [6].

This paper is devoted to the study of the geometric generation of masses in graphene systems as possible source of inspiration for both condensed matter and particle physics. The effective mass of ECCs in graphene is generated by the compactification of a spatial dimension to form CNs. As a consequence of the compactification, the ECCs acquire a rest periodicity which is identified with the Compton periodicity and the resulting effective space-time is 1D with a cyclic character. In undulatory mechanics, through the Planck constant, a finite rest periodicity corresponds to a non vanishing rest mass. This correctly describes the effective mass scale of the carriers in CNs and other systems such as graphene bilayers, quantum stripes and multi-walled CNs. From this Compton periodicity it is possible to infer the effective space-time recurrence (de Broglie—Planck—Einstein space-time periodicities) associated to the motion of the ECCs wave function along a CN. In turns, the space-time recurrence of ECCs imposed as constraint determines the effective energy bands of CNs.

The central result of this paper is that the description of particles as “periodic phenomena”, i.e. as waves with the constraints of Compton periodicity which in CNs is represented by the curled up dimension, permits an intuitive description of several quantum properties of the ECCs in solid state systems. Such geometric considerations can be extended to elementary particles and relativistic space-time [1]. The energy bands of ECCs turn out to be the analogous of the energy levels of the elementary particles quantized spectrum prescribed by second quantization. In particular, graphene systems represent a practical example to visualize the elementary space-time cycles proposed in recent papers [1,2,3,11], where it has been proven that actually the constraint of Compton periodicity (or de Broglie—Planck—Einstein space-time periodicities in a generic reference frame) can be regarded as a semiclassical quantization condition for elementary particles. The intrinsically ultrafast cyclic dynamics of elementary particles, which for Dirac particles correspond to the *zitterbewegung*, have thus a central importance to test the foundations of quantum mechanics. In CNs the Compton time of electrons is rescaled from 10^{-21} s to 10^{-15} s, i.e. to time scale which has recently become accessible to direct experimental observations [16]. Moreover, the approach proposed in this paper represents a simple geometrical description of the ultrafast cyclic dynamics of ECCs in CNs. Graphene-based systems have a rapidly growing interest as ultrafast electronic devices, in which the undulatory nature of ECCs plays a crucial role. Thus an efficient modelization of the fast cyclic dynamics of ECCs is important to control the properties of these devices and the related electronic manipulation of information.

We have also pointed out that the interesting dualism between CNs and extra-dimensional theories can be understood in terms of the Compton recurrence of elementary particles [1,3,11,13] and it shows fundamental analogies with Holography and the AdS/CFT correspondence.

In a recent paper [4] we have applied the geometric considerations about space-time recurrences to derive the basic aspects of superconductivity phenomenology. Concerning CNs, this for instance leads to an heuristic geometric interpretation of the prediction that the critical temperature of superconductivity is inversely proportional to the diameter C_h of the CNs [33]. Since superconductivity, describing an electromagnetic gauge symmetry breaking, is at the base of the Higgs mechanism we may wonder whether our description brings new conceptual elements for a possible geometrodynamical interpretation of the Higgs mechanism, i.e. a description closer to general relativity where masses have actually a geometric meaning in terms of space-time metric.

In a forthcoming paper we will extend the present study to test through graphene systems the geometrodynamical description of gauge interactions obtained in [2]. It is an experimental fact that deformations of the graphene lattice generates a (pseudo) gauge field [34,35]. We will see that this not completely understood phenomenon has a straightforward interpretation in terms of modulations of the quantum recurrence associated to the deformed geometry of the graphene effective space-time. Beside the straightforward computation of the effective magnetic field, this description is particularly relevant because it pinpoints a geometrodynamical nature of gauge interactions, namely a description of gauge interaction based on space-time geometrodynamics similar to those of general relativity. The details of such a general formalism applied to particle physics is given in [2].

The motivation of this and future studies, as well as similar studies present in literature, is to show how graphene physics can be used to probe in “test-tubes” the geometrodynamical origin of elementary particles fundamental prop-

erties such as mass, spin, gauge invariance and gauge symmetry breaking. The interplay between graphene physics and particle physics pointed out in this paper and summarized in Tab.(1) has a general interest for the comprehension of the inner geometric nature of relativistic space-time and its interplay with ordinary matter [1].

References

1. D. Dolce, Europhys. Lett. **102**, 31002 (2013).
2. D. Dolce, Annals Phys. **327**, 1562 (2012).
3. D. Dolce, Annals Phys. **327**, 2354 (2012).
4. D. Dolce, A. Perali, *Quantum recurrence in the electromagnetic gauge symmetry breaking of superconductivity* (2013), [arXiv:1307.5062](#), submitted to Found. Phys.
5. A.H. Castro Neto, F. Guinea, N.M.R. Peres, K.S. Novoselov, A.K. Geim, Rev. Mod. Phys. **81**, 109 (2009).
6. M. Mecklenburg, B.C. Regan, Phys. Rev. Lett. **106**, 116803 (2011).
7. L. de Broglie, Ann. de Physique **3**, 22 (1925).
8. T.M. Rusin, W. Zawadzki, Phys. Rev. B **80**, 045416 (2009).
9. R. Penrose, *Cycles of Time. An Extraordinary View of The Universe* (Knopf, New York, 2011), chap. 2.3.
10. S.Y. Lan, P.C. Kuan, B. Estey, D. English, J.M. Brown, M.A. Hohensee, H. Müller, Science **339**(6119), 554 (2013).
11. D. Dolce, Found. Phys. **41**, 178 (2011).
12. J. Zaanen, Nature Physics **10** 609-610 (2013).
13. D. Dolce, PoS (ICHEP2012), **478** (2013).
14. K. Zou, X. Hong, J. Zhu, Phys. Rev. B **84** 085408 (2011).
15. A. Perali, D. Neilson, A.R. Hamilton, Physical Review Letters **110**(14), 146803 (2013).
16. H. Margolis, Nature Physics **2** 82-83 (2014).
17. K.S. Novoselov, V.I. Fal'ko, L. Colombo, P.R. Gellert, M.G. Schwab, K. Kim, Nature **490**(7419), 192 (2012).
18. J.C. Charlier, X. Blase, S. Roche, Rev. Mod. Phys. **79**, 677 (2007).
19. L. Nottale, *Scale relativity and fractal space-time* (London, UK: Imp. Coll. Pr., 2011).
20. I. Kenyon, *General relativity* (Oxford Science Publications, 1990)
21. J. de Woul, A. Merle, T. Ohlsson, Phys. Lett. B **714**, 44 (2012).
22. A. Perali, A. Bianconi, A. Lanzara, N.L. Saini, Solid State Communications **100**, 181 (1996).
23. J. Scherk, J.H. Schwarz, Nuclear Physics B **153**(0), 61 (1979).
24. Y. Hosotani, Physics Letters B **126**(5), 309 (1983).
25. N. Arkani-Hamed, A.G. Cohen, H. Georgi, Phys. Rev. Lett. **86**, 4757 (2001).
26. D.T. Son, M.A. Stephanov, Phys. Rev. D **69**, 065020 (2004).
27. E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
28. N. Arkani-Hamed, M. Porrati, L. Randall, JHEP **08**, 017 (2001).
29. A. Karch, E. Katz, D.T. Son, M.A. Stephanov, Phys. Rev. D **74**, 015005 (2006).
30. A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, V.F. Weisskopf, Phys. Rev. D **9**, 3471 (1974).
31. M. Neubert, Phys. Lett. B **660**, 592 (2008).
32. M.A. Stephanov, Phys. Rev. D **76**, 035008 (2007).
33. J.W.M. C. T. White, Nature **6688**, 29 - 30 (1998).
34. M.A.H. Vozmediano, M.I. Katsnelson, F. Guinea, Physics Reports **496**, 109 (2010).
35. A. Iorio, J. Phys. Conf. Ser. **442** 012056 (2013).